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RESEARCH MEMORANDUM

A METHOD FOR INCREASING THE EFFECTIVENESS OF STABILIZING

SURFACES AT HIGH SUPERSONIC MACH NUMBERS

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A METHOD FOR INCREASING THE EFFECTIVENESS OF STABILIZING

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SUMMARY

Large increases in lift-curve slope at low angles of attack at high supersonic Mach numbers can be obtained by the use of wedge-shape airfoil sections. The use of such sections on the tail surfaces operating at low angles of attack on airplanes or missiles traveling at these speeds can greatly decrease the stabilizing-surface area required. Moderate increases in section effectiveness (lift-curve slope) over that for more conventional tail sections can be obtained with little or no increase in total drag because of the decreased surface area required. Larger increases can be obtained; however, they will be accompanied by increases in stabilizing-surface drag.

INTRODUCTION

A major problem in flight at supersonic speeds is the decrease in lift-curve slope of lifting surfaces with increase in Mach number and, hence, the reduction of stability and control effectiveness. The magnitude of this problem is illustrated by the fact that at a Mach number of 7 the lift-curve slope for thin airfoils at low angles of attack is about one-sixth the value obtained at a Mach number of 1.5. Furthermore, the variation in body force coefficients with Mach number can be an adverse factor in the stability and control of the airplane or missile. Increasing the stability by increasing the number or the area of the stabilizing surfaces and their moment arms can sometimes resolve the problem but may often result in unwieldy and inefficient configurations.

The use of a simple wedge for the stabilizing surface appears to be one way to increase the effectiveness of the surface at high supersonic Mach numbers without increasing the surface area. Small gains in lift-curve slope have been reported in references 1, 2, and 3 by using small wedges instead of sharp-trailing-edge airfoils. However, the magnitude of the increases that can be obtained has not been adequately

discussed, particularly with regard to large wedge angles and high super-
sonic Mach numbers. The purpose of this report therefore is to bring
attention to the large benefits that may be obtained from increasing the
wedge angle of stabilizing surfaces, particularly at high supersonic Mach
numbers.

SYMBOLS

C_D drag coefficient

C_{D_i} inviscid drag coefficient

C_f friction coefficient (both surfaces)

C_L lift coefficient

C_{L_α} lift-coefficient-curve slope

L lift

c chord

p static pressure

p_L static pressure on lower surface

p_U static pressure on upper surface

p_b base pressure

t airfoil thickness

M Mach number

x distance along chord

α angle between free-stream flow and chord line

δ flow deflection angle

Subscripts:

1   conditions ahead of shock

2   conditions behind shock

o   condition at $\alpha = 0^\circ$

BASIC CONSIDERATIONS

A stabilizing surface must be able to produce a large restoring moment when it is disturbed from alinement with the stream. (To simplify the discussion, the following is confined to flat plates and airfoils without camber.) For a given moment arm, the value of $C_{L_{\alpha_O}}$ determines the effectiveness of a surface operating at low angles of attack in producing a restoring moment.

The lift coefficient on an airfoil section can be expressed as

$$C_L = \frac{1}{c} \int_0^c \frac{p_L - p_U}{q}\, dx$$

where p_L and p_U are the pressure over the lower and upper surfaces.

At zero angle of attack the rates of changes of $p_L - p_1$ and $p_U - p_1$ with angle of attack for a symmetrical airfoil may be assumed equal but opposite in sign. It can be shown that for a flat plate

$$C_{L_{\alpha_O}} = \frac{4}{\gamma M^2} \frac{d\left(\dfrac{p_2}{p_1}\right)}{d\alpha}$$

where p_2/p_1 is the ratio of the pressure behind the shock to the free-stream pressure. For a symmetrical airfoil other than the flat plate the average pressure $\bar{p}_2$ over the surface may be substituted for p_2.

For a flat plate $\dfrac{d\left(\dfrac{p_2}{p_1}\right)}{d\alpha}$ at $\alpha = 0^O$ is equal to $\dfrac{d\left(\dfrac{p_2}{p_1}\right)}{d\delta}$ where δ is the flow deflection angle. Above a Mach number of about 2, $\dfrac{d\left(\dfrac{p_2}{p_1}\right)}{d\delta}$ for a flat plate is approximately proportional to the Mach number so that $C_{L_{\alpha_O}}$ is approximately inversely proportional to M.

Figure 1, which presents graphically the well known relation

between p_2/p_1 and δ, shows that the slopes of these curves $\dfrac{d\left(\dfrac{p_2}{p_1}\right)}{d\delta}$

increases rapidly as δ is increased, particularly at the higher Mach
numbers. By adding thickness, as in a wedge airfoil section, the δ
of the surfaces can be increased at a constant angle of attack, thereby

giving a larger value of $\dfrac{d\left(\dfrac{p_2}{p_1}\right)}{d\delta}$, and consequently a larger value of $C_{L_{\alpha_0}}$.

The ratio of the slope $\dfrac{d\left(\dfrac{p_2}{p_1}\right)}{d\delta}$ at any deflection angle to that at $\delta = 0^{\circ}$

is the ratio of the effectiveness of the surface to that of a flat plate
at zero angle of attack. The use of a wedge airfoil is the most effec-
tive way of increasing the section effectiveness since the deflection
angle is then constant over the surfaces.

WEDGE AIRFOIL CHARACTERISTICS

Effect of Wedge Angle and Mach Number on Lift-Curve Slope

The variation of the section lift-curve slope with Mach number for
various wedge half-angles as obtained from the slopes of curves similar
to those in figure 1 is presented in figure 2. The rapid decrease in
the lift-curve slope for a flat plate, $\delta = 0^{\circ}$, is readily apparent in
this plot. This figure shows that very large gains can be obtained in
section lift-curve slope with moderately small wedge half-angles. From
this figure it is possible to determine the wedge angles required to
maintain a constant value of $C_{L_{\alpha_0}}$ as Mach number is varied. The lift-
curve slope for a 5-percent-thick diamond airfoil section (which has a
wedge half-angle of 2.86°) is also included in figure 2 to show the
variation for a more conventional wing section.

It must be pointed out that these values are inviscid section
values. At high Mach numbers the presence of the boundary layer will
increase the pressures over the surface (refs. 3 and 4) in such a way
as to increase the lift-curve slope slightly.

In an actual application of these calculations, plan-form and wing-
body interference effects must also be considered. Investigations at
$M = 6.9$ (refs. 3, 4, and 5) have shown that as long as the shock is
attached to the wing leading edge the losses in lift due to plan-form
effects are small. Unpublished results obtained in the Langley 11-inch
hypersonic tunnel have shown that as would be expected the interference

effects are small at Mach numbers of the order of 7. However, investigations of interference and plan-form effects for the particular configuration are advisable.

The ratio of the stabilizing-surface area required to obtain a given stabilizing force for a wedge to that required for a flat plate is equal to the reciprocal of the ratio of the lift-curve slopes of the two surfaces at $\alpha = 0^O$. This ratio of lift-curve slopes is therefore presented in figure 3 as a measure of the effectiveness of a stabilizing surface. It is interesting to note that for Mach numbers above 4 the value of $\dfrac{C_{L_{\alpha_O}}}{C_{L_{\alpha_O}(\text{flat plate})}}$ is, for all practical purposes, directly proportional to Mach number. Values of relative surface effectiveness of about 4.4 are obtainable at $M = 7$ with the 15° wedge half-angle. Similar curves can be constructed for wedge half-angles up to the shock detachment angle (40° to 45° at Mach numbers above 4.5) with far greater values of $\dfrac{C_{L_{\alpha_O}}}{C_{L_{\alpha_O}(\text{flat plate})}}$.

Effect of Wedge Angle and Mach Number on Drag

The large increases in lift-curve slope are not accomplished without some penalty. For example, the inviscid section drag with the large-angle wedges is much higher than for more conventional sections such as a 5-percent-thick double-wedge section with the maximum thickness at 50 percent chord. Figure 4 shows the inviscid drag coefficients for several wedge angles plotted against Mach number for two conditions of base pressure. As shown, the magnitude of the base pressure is not important at the extremely high Mach numbers. No attempt has been made to evaluate the base pressure.

It may be seen from figure 4 that the increases in inviscid drag with wedge angle are large. However, skin friction should be included in any drag comparison. Furthermore, since the surface area can be decreased for constant lift because of the increased $C_{L_{\alpha_O}}$ with increasing δ, the area reduction with increasing wedge angle should be taken into account. Therefore, in figure 5 the total drag of the wedge has been divided by the dynamic pressure and the flat-plate area (which is held constant). The area of the wedge airfoil has been decreased so that the lift and consequently the restoring moment per degree will be constant. This curve therefore indicates the drag penalty that results from the use of wedge surfaces. The ordinate on this figure is equal

to the drag coefficient based on the actual area of the wedge section

divided by $\dfrac{C_{L_{\alpha_O}}}{C_{L_{\alpha_O}(\text{flat plate})}}$. The calculations have been made for a

laminar flat-plate skin-friction coefficient of 0.00217 (two sides). The skin friction on the wedge was corrected for the difference in Reynolds number, Mach number, and dynamic pressure ahead and behind the shock. The calculated skin-friction coefficient including these corrections increases with wedge angle. Since a laminar flow is considered, the base pressure will probably be near free-stream pressure. A base pressure equal to stream pressure has therefore been assumed in this figure.

Also included in this figure is the value for a 5-percent-thick diamond section. For the particular conditions selected for this comparison the total drag of the diamond section is equal to that for the wedge with $\delta = 4^{\circ}$ and the greater effectiveness of the wedge $\left(C_{L_{\alpha_O}}\text{ is }\right.$

1.61 times the $C_{L_{\alpha_O}}$ of the diamond section$\left.\right)$ results in a tail surface with only 62 percent the area required with a 5-percent-thick diamond section. Below $\delta = 4^{\circ}$ the wedge section has less total drag than the diamond section and would still have a large advantage in size as well as reduced drag at lower Mach number if the wedge angle is variable with Mach number. In the actual selection of wedge angle, calculations should be made over the complete Mach number and Reynolds number range of operation and a determination of the base pressure for the operating range should be made.

APPLICATION

In application, the wedge angle might be varied with Mach number as shown in figure 6. A variable angle wedge would allow $C_{L_{\alpha_O}}$ to be continuously adjustable in flight so as to allow the stability to be varied as needed and would allow low drags to be obtained at Mach numbers below the maximum design Mach number because use of a variable wedge angle allows the use of reduced surface area.

The large advantages of the wedge surface can only be realized when the stabilizing surface is operating at low angles with the stream. If the surface must operate at large angles of attack in order to supply sufficiently large lift coefficients to maintain trim, a simple flat plate will have already gained much of the effectiveness of the wedge. Therefore, the usefulness of the wedge surface is mainly in configurations where trim is obtained at low angles, such as the vertical tail surface or all the tail surfaces of a configuration traveling in nearly a ballistic trajectory.

The reduction in size of the tail surface may be important from the standpoint of weight as well as space required to house the missile or airplane. The weight reduction possible with the use of wedge sections, which may be considerably greater than the area reduction, can be a very important factor in many cases, particularly since a forward shift in center of gravity due to the reduced tail weight will further decrease the stabilizing surfaces required.

CONCLUDING REMARKS

Large increases in lift-curve slope at high supersonic Mach numbers can be obtained by the use of wedge-shape airfoil sections. For example, 4.4 times the flat-plate effectiveness (lift-curve slope) can be obtained with a surface using a wedge half-angle of 15° at a Mach number of 7. The use of such sections on the tail surface of airplanes or missiles traveling at such high speeds can greatly decrease the stabilizing-surface area required. The effectiveness will increase up to the point of shock detachment (between 40° and 45° above a Mach number of 4.5 for a two-dimensional surface). In general, the large increase in section effectiveness will be accompanied by large increases in stabilizing-surface drag. However, moderate increases in section effectiveness over that of more conventional tail sections can be obtained with little or no increase in total drag since the greater effectiveness allows a reduction in the area required to maintain a stable configuration.

The usefulness of the wedge surface is mainly in configurations where trim is obtained at low angles of attack on the stabilizing surfaces since a surface such as a flat plate which operates at the higher angles of attack will possess much of the effectiveness of a surface with a wedge section.

Langley Aeronautical Laboratory,
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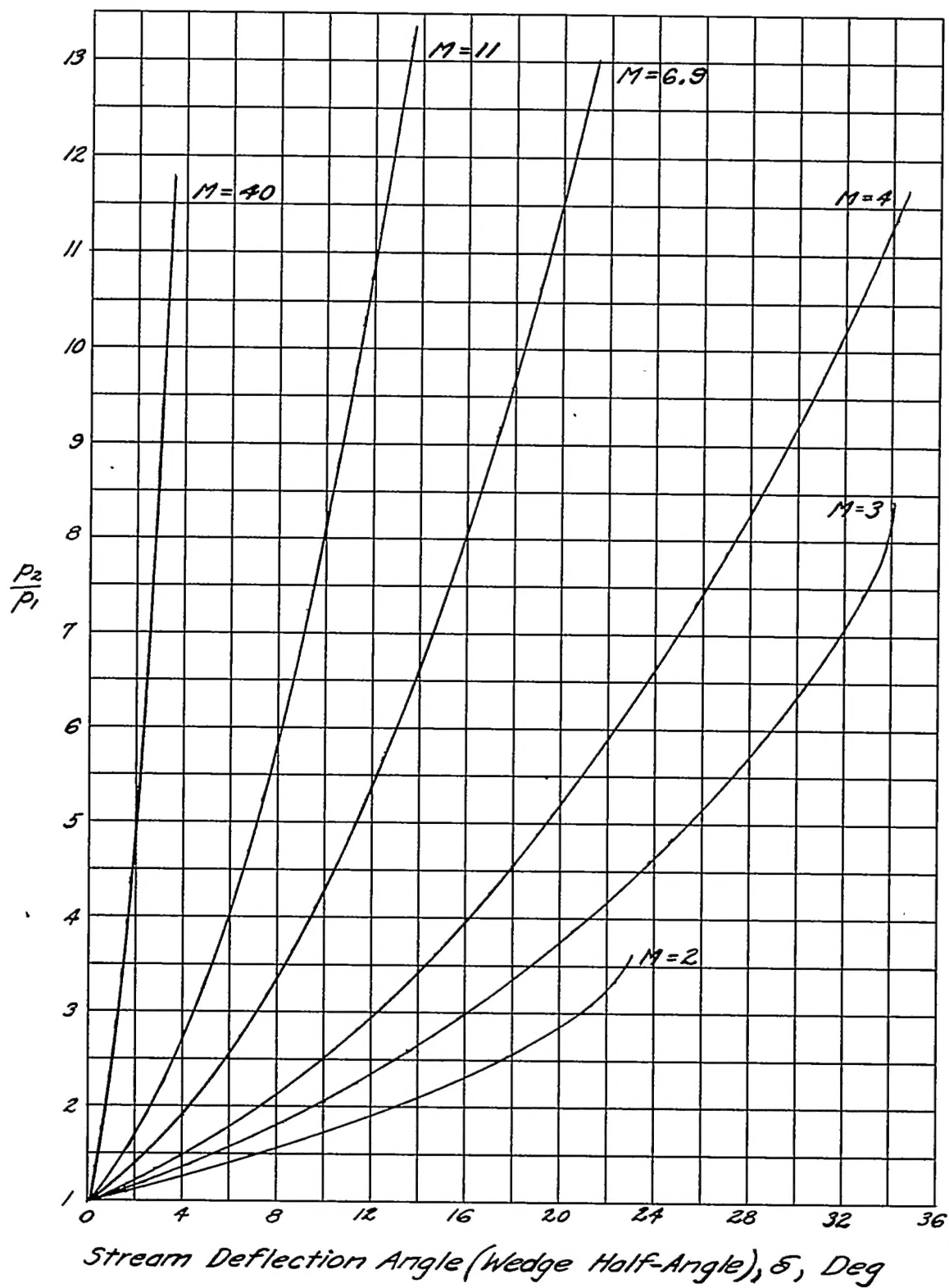
Figure 1.- Variation of pressure rise across shock for various Mach numbers and deflection angles. $\gamma = 1.4$.
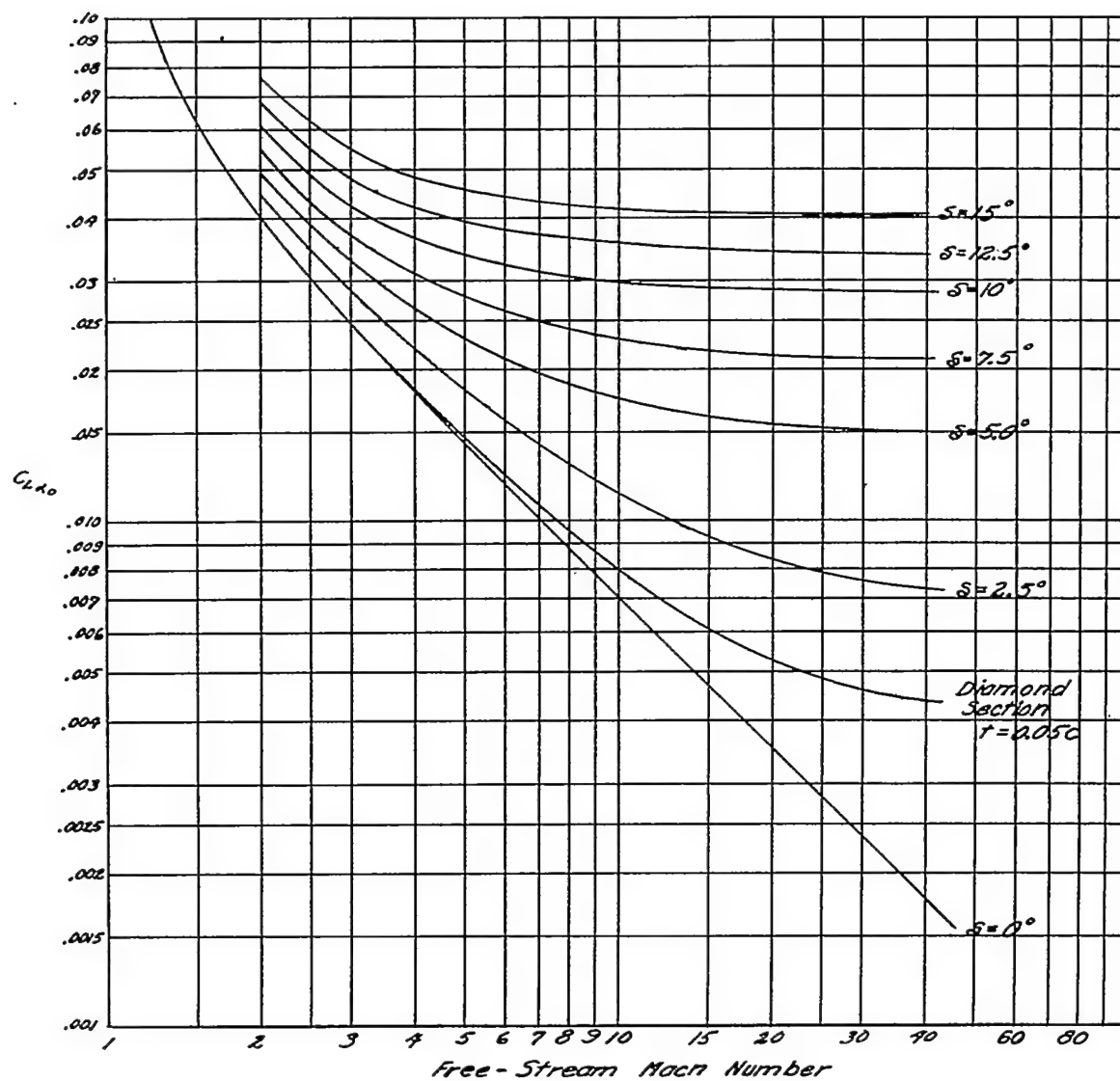
Figure 2.- Variation of lift-curve slope at $\alpha = 0^\circ$ with Mach number for various wedge half angles.
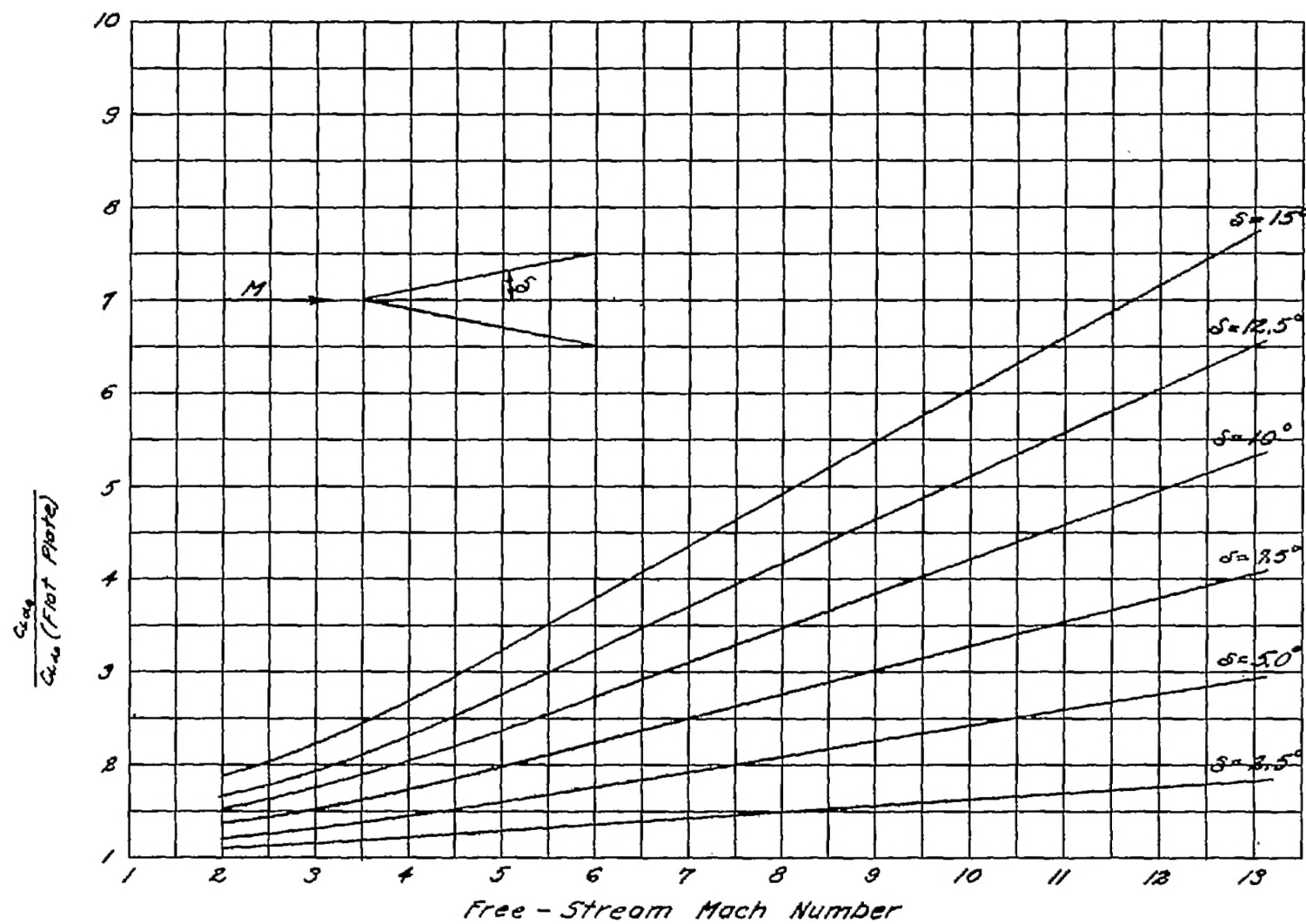
Figure 3.- Effect of wedge angle and Mach number on ratio of lift-curve slope of wedges to that for a flat plate at zero angle of attack.
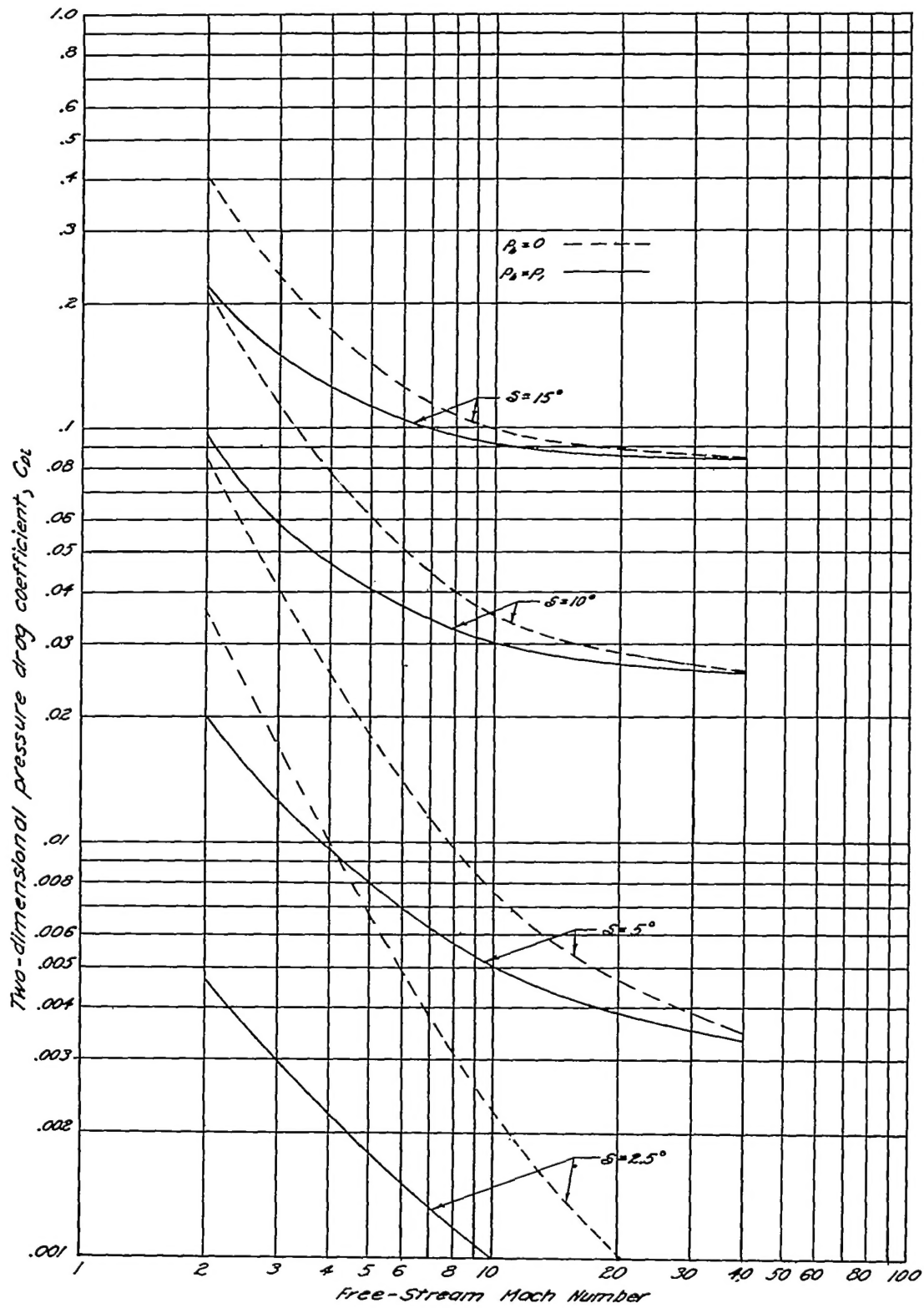
Figure 4.- Variation of inviscid drag coefficient of wedges with Mach number.
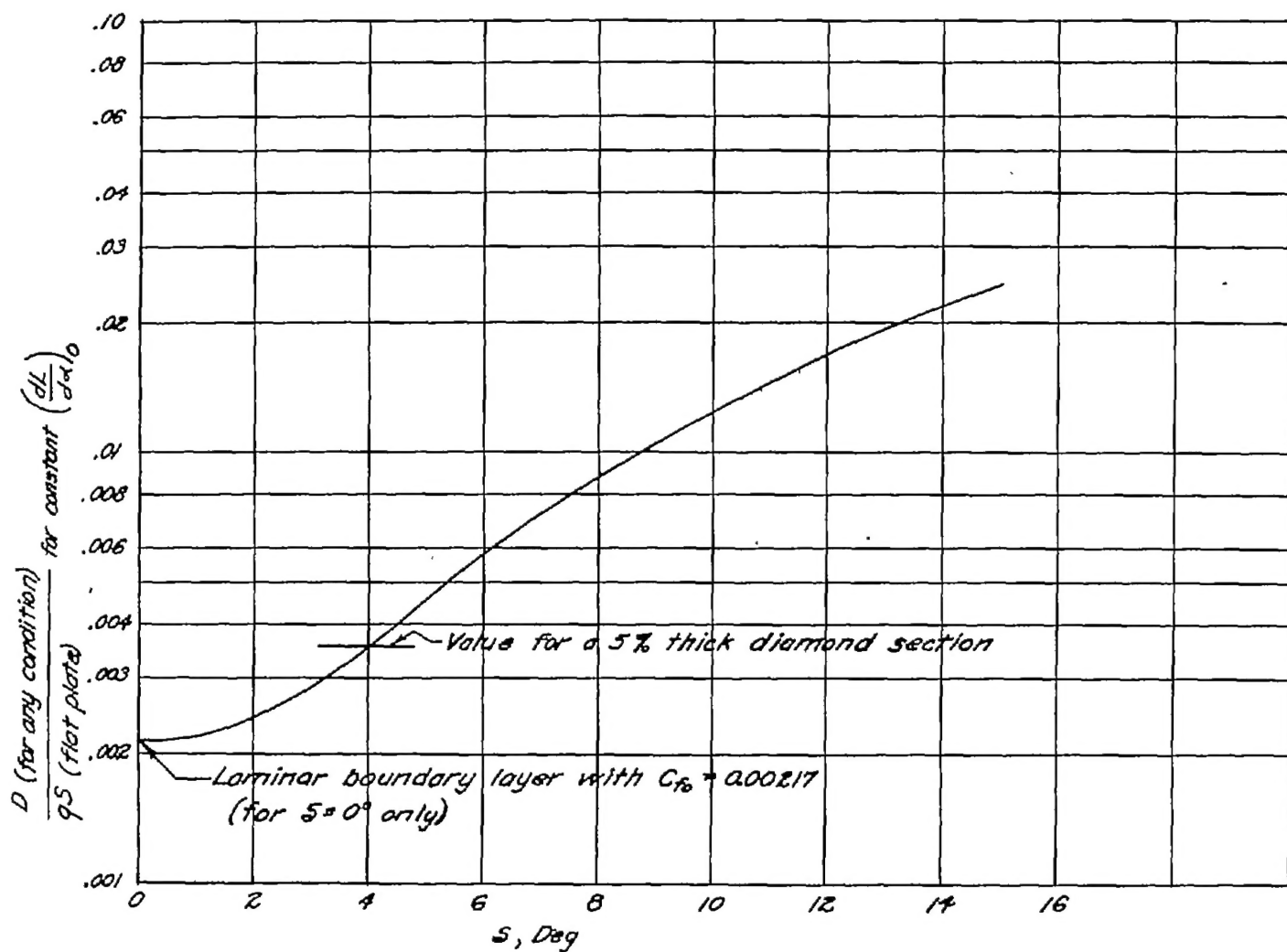
Figure 5.- Variation of drag for a given lift-curve slope at $\alpha = 0^\circ$ and $M = 6.9$.
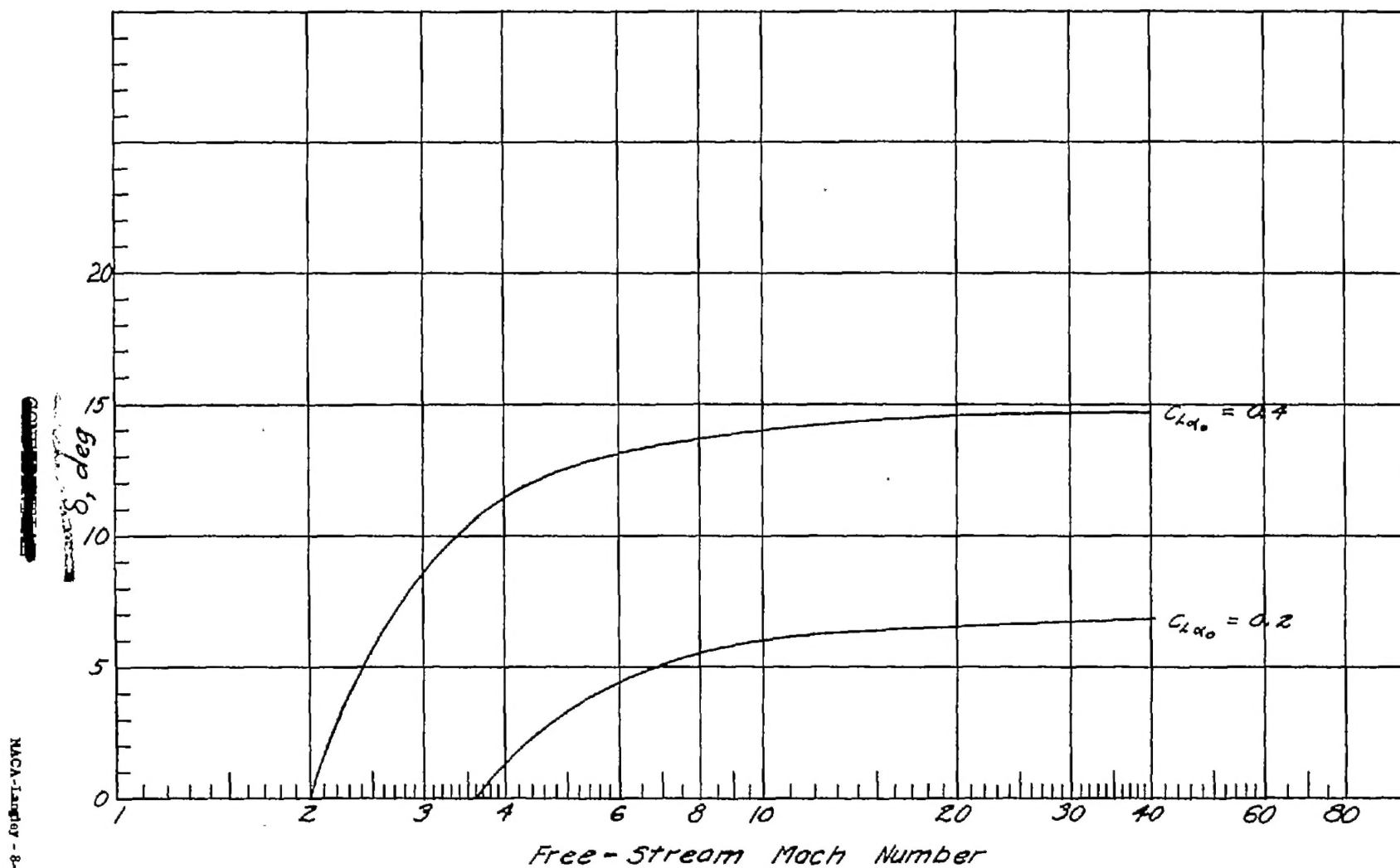
Figure 6.- Variation of wedge half angle with Mach number for two lift-
curve slopes.